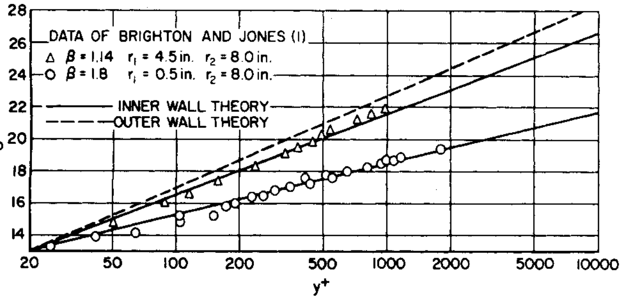

Fig. 3. Comparison of experimental data with proposed theories.

Figure 3 shows a comparison of the experimental data of Brighton and Jones (*1*) with the proposed theory for two values of β. As shown in the figure, there is a very good agreement between the data and the theoretical distribution.

NOTATION

n = parameter defined in Equation (2)
r = radial distance, ft.
r_m = radius of maximum velocity, ft.
U = time-averaged local axial velocity, ft./sec.
U_τ = friction velocity, ft./sec.
U^+ = dimensionless velocity, u/u_r
y = distance from wall, ft.
y^+ = dimensionless distance, $y\ u_r/v$

Subscripts

1 = inner surface of the annulus
2 = outer surface of the annulus
m = portion of maximum velocity

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On the Height of Incipience of Boiling in Tubes of Small Diameter for a Liquid in Laminar Flow

WAYNE J. DRDA and ERIC WEGER

Washington University, St. Louis, Missouri

One of the factors influencing the overall rate of heat transfer to a liquid flowing in a tube is the point (if any) where nucleate boiling begins. This problem has been considered by Davis and Anderson (*1*) for a turbulent flow situation where the bubble nuclei develop within the laminar sublayer where linear temperature profiles can be assumed. However for tubes of small diameter, such as are used in some nuclear and aerospace applications, the liquid is often in laminar flow and there is need for a technique to predict incipience of boiling for this flow regime. Siegel, Sparrow, and Hallman (*2*) have obtained the solution for the temperature distribution in a liquid flowing in a vertical tube (at positions sufficiently removed from the entrance to allow the neglect of entrance effects) of constant radius. Liquid properties were assumed to be independent of temperature and a constant heat flux at the wall was assumed.

$$\frac{T-T_o}{(qr_o/k)} = \frac{4(z/r_o)}{N_{Re}N_{Pr}} + \left(\frac{r}{r_o}\right)^2 - \frac{1}{4}\left(\frac{r}{r_o}\right)^4 - \frac{7}{24} + \sum_{n=1}^{\infty} C_n\, e^{-\frac{\beta_n^2}{N_{Re}N_{Pr}}\left(\frac{z}{r_o}\right)} R_n\left(\frac{r}{r_o}\right) \qquad (1)$$

The values of C_n, β_n, and $R_n(r/r_o)$ were obtained numerically and the first seven values of C_n and β_n are given in their paper. Tables of the first four eigenfunctions, $R_n(r/r_o)$, were obtained from Siegel (*3*).

By using the commonly accepted theory that the bubbles originate at small cavities in the system walls, one can write the following equation for the segment of a spherical bubble at the wall at equilibrium.

$$P_g - P_l = \frac{2\sigma}{r_b} \qquad (2)$$

If the vapor is assumed to be a perfect gas, the Clausius-Clapeyron equation can be used to replace the vapor pressure in Equation (2) with temperatures, to obtain

$$T_g - T_s = \left(\frac{T_g T_s R}{\lambda}\right) \ln\left(1 + \frac{2\sigma}{r_b P_1}\right) \qquad (3)$$

Assuming that the bubble at the start of its growth is a hemisphere and following the reasoning of Bergles (*4*), the following conditions must be met before the bubble will grow.

$$T_g = T \text{ at } r_b = y$$

$$\frac{\partial T_g}{\partial r_b} = \frac{\partial T}{\partial y} \text{ at } r_b = y$$

$$y = r_o - r$$

Applying these conditions to Equations (1) and (3) and rearranging them slightly, one obtains the following set of equations

$$1 + Ki\left[\frac{4}{N_{Re}N_{Pr}}\left(\frac{h}{r_o}\right) + \left(1 - \frac{r_b}{r_o}\right)^2 - \frac{1}{4}\left(1 - \frac{r_b}{r_o}\right)^4 - \frac{7}{24}\right.$$

Wayne J. Drda is with the U.S. Army at Fort Bliss, Texas.

$$+\sum_{n=1}^{\infty} C_n\, e^{-\frac{\beta_n^2}{N_{Re}N_{Pr}}\left(\frac{h}{r_o}\right)} R_n\left(\frac{r_b}{r_o}\right)\Bigg]$$

$$-\left(\frac{T_s}{T_o}\right)\left[\frac{1}{1-(H)\ln\left(1+\dfrac{N_{We'}}{r_b/r_o}\right)}\right]=0 \qquad (4)$$

and

$$2\left(1-\frac{r_b}{r_o}\right)-\left(1-\frac{r_b}{r_o}\right)^3$$

$$-\sum_{n=1}^{\infty} C_n\, e^{-\frac{\beta_n^2}{N_{Re}N_{Pr}}\left(\frac{h}{r_o}\right)} \frac{d\, R_n(r_b/r_o)}{d\,(r_b/r_o)}$$

$$-\frac{\left(\dfrac{T_s}{T_o}\right)(H)\,(N_{We'})}{N_{Ki}}$$

$$\frac{\dfrac{r_o^2}{r_b}\left(\dfrac{1}{1+[N_{We'}/(r_b/r_o)]}\right)}{\left[1-(H)\ln\left(1+\dfrac{N_{We'}}{r_b/r_o}\right)\right]^2}=0 \qquad (5)$$

Equations (4) and (5) were solved numerically for h/r_o and r_b/r_o for various values of N_{Re}, N_{Pr}, N_{Ki}, $N_{We'}$, H, and T_s/T_o. The calculations were made using Newton's Method on an IBM 7072 computer (5).

Once h/r_o has been determined, the wall temperature can be calculated from Equation (1) by setting $r = r_o$.

RESULTS

Equations (4) and (5) were solved for h/r_o and r_b/r_o for the following values of the dimensionless parameters.

$$50 \leq N_{Re}\,N_{Pr} \leq 7{,}500$$
$$0.001 \leq N_{Ki} \leq 2.5$$
$$0.000365 \leq N_{We'} \leq 0.0234$$
$$0.0022 \leq H \leq 0.22$$
$$1.06 \leq T_s/T_o \leq 1.63$$

By considering the variation of h/r_o with each of these quantities, the following relationships were developed.

For $N_{Ki} \leq 0.01$

$$\frac{h}{r_o} = \psi\left[\frac{N_{Re}N_{Pr}}{N_{Ki}}\right] \qquad (6)$$

where

$$\psi = -0.2520 + 0.2505\left(\frac{T_s}{T_o}\right) \qquad (7)$$

For $N_{Ki} > 0.01$

$$\frac{h}{r_o} = N_{Re}\,N_{Pr}\,\exp\{\ln[\psi\,(N_{Ki})^{-1}] - \epsilon\} \qquad (8)$$

No simple analytical function could be found which would accurately express ϵ as a function of N_{Ki}, $N_{We'}$, and T_s/T_o. Figure 1 presents typical ϵ vs. N_{Ki} data. Graphs presenting ϵ for other T_s/T_o ratios are available in reference (5).

The dimensionless group H was found to have a negligible effect on h/r_o for all values of the other parameters used.

In view of the assumption of a hemispherical bubble at the start of its growth, r_b represents the radius of the cavity which served as the nucleating site. The ratio r_b/r_o was found to be independent of both N_{Re} and N_{Pr}. It is a function of the other four dimensionless groups. It is not strongly affected by T_s/T_o. Unlike h/r_o, it does not vary with H. The group r_b/r_o was not considered important enough to justify the time required to develop an equation expressing it as a function of the dimensionless groups as was done for h/r_o.

DISCUSSION

The method of calculating h/r_o, the point of incipient boiling, does not take into consideration the nucleating cavity directly. Obviously, if boiling is to begin at the height calculated there must be a nucleating cavity of the proper dimensions at this point. This is not as severe a restriction as one might think since most commercially prepared surface have numerous nucleating sites of all sizes and shapes. The neglect of nucleating sites in deriving Equations (6) and (8) can therefore not be expected to seriously affect the results. However, this method would not be expected to apply to any system which had been treated in such a way that only cavities of a particular size distribution remained.

The value of h/r_o calculated by this method gives the lower limit of the height for incipience of boiling. This is shown in Figure 2 where the point that is calculated is indicated. Any intersection of the solid curve with the dashed curves that lies to the left of this point would be an effective nucleating site. For these points the following conditions apply:

$$T = T_g$$
$$\frac{\partial T}{\partial y} > \frac{\partial T_g}{\partial r_b}$$

That is, the bubble would grow into a liquid superheated with respect to its vapor temperature. However these intersections all have values of h/r_o greater than that given by Equations (6) or (8). Therefore, since the concern here is with vertical tubes of small diameter, the bubbles

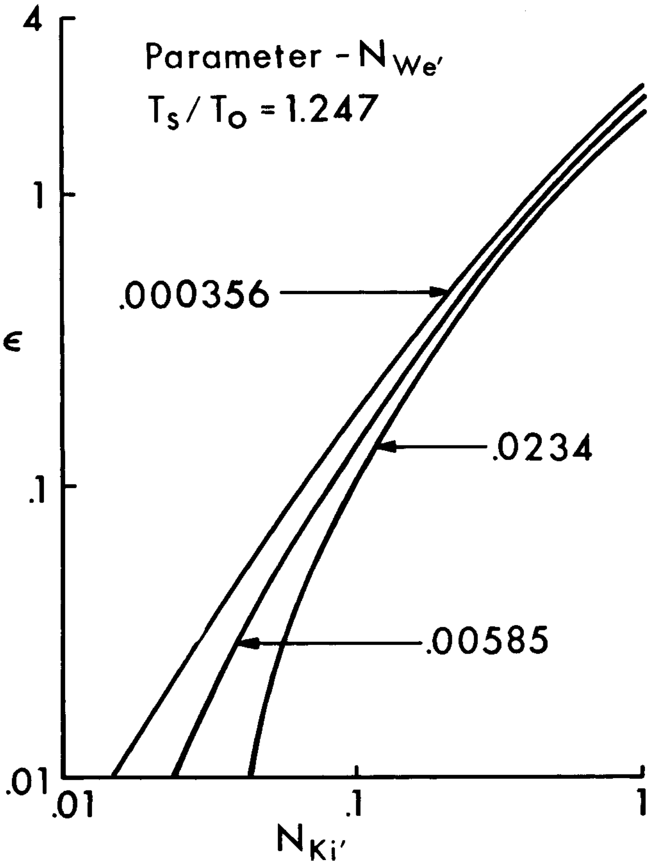

Fig. 1. Correction factor.

formed below these points probably would occupy enough of the cross-sectional area as they rose so that they would wipe off any incipient bubbles at the larger values of h/r_o. Movies taken of boiling water in a 0.635 cm. diam. tube have shown this to be the case. However, some of these sites might be effective if the proper size nucleating cavity did not exist at the h/r_o given by Equation (6) or (8).

Those intersections lying to the right of the indicated point would not be effective nucleating sites since in this case any bubbles formed would be growing into a subcooled liquid and would eventually collapse. It is possible that the bubble could grow into the subcooled liquid up to a point where the evaporation near the base of the bubble just equaled the condensation near the top. This condition, however, would not be very stable since any disturbance of the flow, such as that caused by other bubbles growing and collapsing, would upset this balance. An excellent discussion of this is given by Ellion (*6*).

As shown in Figure 3 as N_{Ki} decreases, r_b/r_o increases, eventually approaching one. At this point since $r_b = r_o$, the bubble at the start of its growth would fill a large part of the tube. This would obviously have a notable effect on the velocity and flow field in the vicinity of the nucleating site and therefore on the bubble itself. A more complicated analysis than the one presented here would be required to take these effects into consideration. The present results would not be expected to apply in this region.

Throughout this development it has been assumed that the bubble at the start of its growth was hemispherical in shape. However, Equation (2) is equally applicable to a nucleated bubble which is any part of a sphere and the method used is still valid even if the assumption of a hemisphere is not. The point at which the conditions given by Bergles (*4*) apply would have to be changed and r_b would no longer represent the radius of the nucleating cavity.

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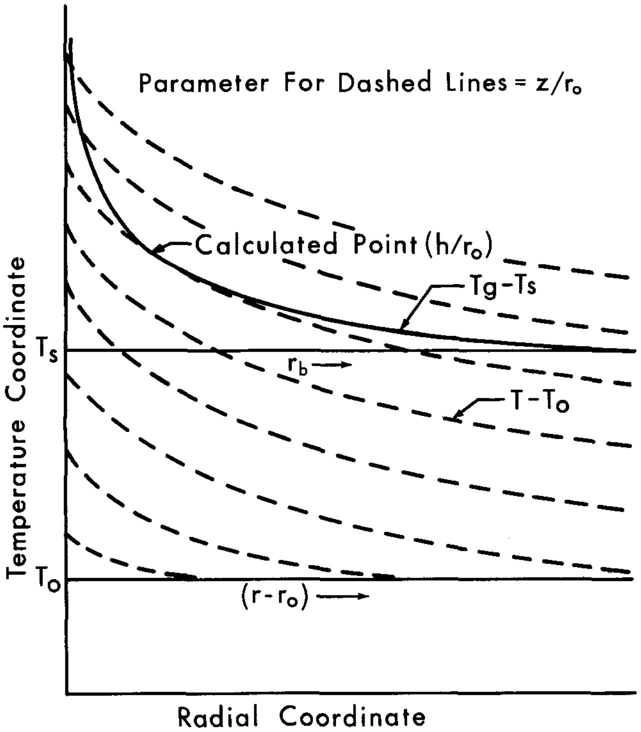

Fig. 2. Graphical representation of the analytical solution.

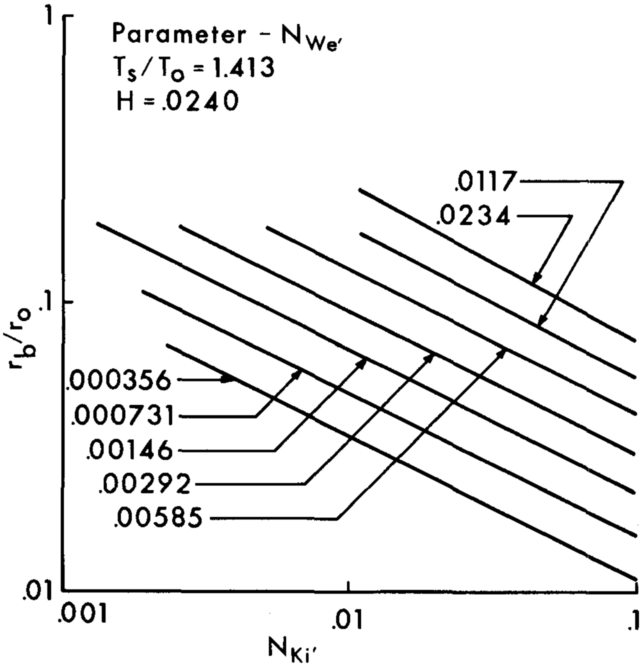

Fig. 3. Variation of r_b/r_o with N_{ki}.

NOTATION

c_p = heat capacity at constant pressure of the liquid
H = dimensionless number, T_sR_g/λ
h = height of incipience of boiling
k = liquid thermal conductivity
N_{Ki} = modified Kirpichev number, qr_o/kT_o
N_{Pr} = Prandtl number, $c_p\mu/k$
P_g = vapor pressure inside nucleated bubble
P_1 = pressure in liquid surrounding nucleated bubble
q = heat flux at the tube wall
R = universal gas constant
N_{Re} = liquid Reynolds number, $2r_o \langle v_a \rangle \rho/\mu$
r = radial coordinate in cylindrical system
r_b = radius of nucleated bubble at incipience of boiling
r_o = tube radius
T = temperature
T_g = temperature of vapor inside nucleated bubble
T_o = temperature of the liquid at the tube entrance
T_s = saturation temperature of liquid
$\langle v_a \rangle$ = average liquid velocity
$N_{We'}$ = modified Weber number, $2\sigma/r_oP_1$
y = $r_o - r$
z = axial coordinate in cylindrical coordinate system
λ = heat of vaporization
μ = liquid viscosity
ρ = liquid density
σ = surface tension

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